

Mathematics!



A Story of Units Parent Handbook

Grade 2 Module 6

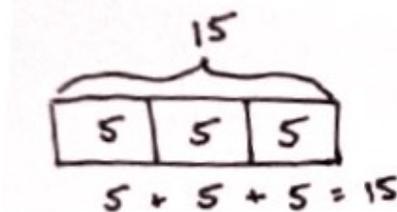
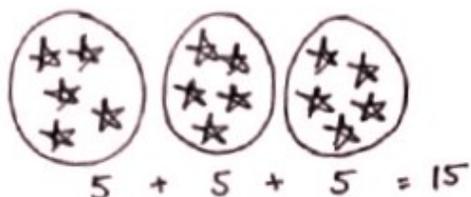
Grade 2 • Module 6

Foundations of Multiplication and Division

OVERVIEW

Grade 2 Module 6 lays the conceptual foundation for multiplication and division in Grade 3 and for the idea that numbers other than 1, 10, and 100 can serve as units.

In Topic A, students begin by making equal groups using concrete materials, learning to manipulate a given number of objects to create equal groups (e.g., given 15 objects, they create 3 groups of 5 or 5 groups of 3), and progress to pictorial representations, where they may begin by circling a group of 5 stars, adding 5 more, then adding 5 more. They determine the total and relate their drawings to the corresponding repeated addition number sentence (pictured below). Students calculate the repeated addition sums by adding on to the previous addends, step by step, or by grouping the addends into pairs and adding. By the end of Topic A, students are drawing abstract tape diagrams to represent the total and to show the number in each group as a new unit (pictured below). Hence, they begin their experience towards understanding that any unit may be counted, e.g., 3 dogs, 3 tens, or even 3 fives. This is the bridge between Grades 2 and 3: Grade 2 focuses on the manipulation of place value units, whereas Grade 3 focuses on the manipulation of numbers 1 through 10 as units.



In Topic B, students organize the equal groups created in Topic A into arrays, wherein either a row or column is seen as the new unit being counted. They use manipulatives to compose up to 5 by 5 arrays one row or one column at a time, and express the total via repeated addition number sentences. For example, students might arrange one column of 5 counters, then another, and another to compose an array of 3 columns of 5, or 15 counters. As they compose and decompose arrays, students create different number sentences yielding the same total (e.g., $5 + 5 + 5 = 15$ and $3 + 3 + 3 + 3 + 3 = 15$). They find the total number of objects in each array by counting on from left to right. "Three plus 3 is 6. Six plus 3 is 9. Nine plus 3 is 12." As Topic B progresses, students move to the pictorial level to represent arrays and to distinguish rows from columns by separating equal groups horizontally and vertically (e.g., 3 columns of 5 or 5 rows of 3). Then they use tiles, moving them closer together in preparation for composing rectangles in Topic C. Topic B concludes with students using tape diagrams to represent array situations and the RDW process to solve word problems.

In Topic C, students build upon their work with arrays to develop the spatial reasoning skills they will need in preparation for Grade 3's area content. They use same-size squares to tile a rectangle with no gaps or overlaps and then count to find the total number of squares. After composing rectangles, students partition, or decompose, rectangles: first with tiles, then with scissors, and finally, by drawing and iterating a square unit. In doing so, they begin to see the row or the column as a composite of multiple squares or as a single entity, or unit, which is, in turn, part of the larger rectangle. Students further develop spatial structuring skills by copying and creating drawings on grid paper. Note that the concept of a square unit begins in Grade 3 and is not assessed in Grade 2. Throughout the topic, students relate repeated addition to the model. They are encouraged to think flexibly and to consider the many ways to construct or partition a given array. Students are not multiplying or dividing in Grade 2; rather, this topic lays the foundation for the relationship between the two operations: As equal parts can be composed to form a whole, likewise, a whole can be decomposed into equal parts.

Topic D focuses on doubles and even numbers, thus setting the stage for the multiplication table of two in Grade 3. As students progress through the lessons, they learn the following interpretations of even numbers:

1. A number that occurs as we skip-count by twos, starting with the number two, is even. If we start with 3 and skip count by twos we will generate odd numbers.
2. When objects are paired up with none left unpaired, the number is even.
3. A number that is twice a whole number (doubles) is even.
4. A number whose last digit is 0, 2, 4, 6, or 8 is even.

Armed with an understanding of the term *even*, students learn that any whole number that is not even is called *odd*, and that when 1 is added to or subtracted from an even number, the resulting number is odd.

Initially, students arrange pairs into two rows, and realize that an even number is the sum of two equal addends or a repeated sum of twos. They then write number sentences to express the even number (e.g., 2 rows of 7 can be expressed as $7 + 7$ or as $2 + 2 + 2 + 2 + 2 + 2$). Next, students pair objects to make groups of two with none left over, thus discovering one means of determining whether a group of objects (up to 20) has an even or odd number of members. Finally, they learn that any number up to 20 whose last digit is 0, 2, 4, 6, or 8 is even. After gaining a firm understanding of even numbers, students learn that all other whole numbers are odd. They use the previously learned rules and patterns to identify larger numbers as even or odd and to defend their reasoning. The module concludes with an investigation of what happens when we add two even numbers, two odd numbers, or an odd number with an even number, and their relationship to repeated addition (e.g., $3 + 3$ is even, but $3 + 3 + 3$ is odd).

Terminology

Terminology

New or Recently Introduced Terms

- Array (arrangement of objects in rows and columns)
- Columns (the vertical groups in a rectangular array)
- Even number (a whole number whose last digit is 0, 2, 4, 6, or 8)
- Odd number (a number that is not even)
- Repeated addition (e.g., $2 + 2 + 2$)
- Rows (the horizontal groups in a rectangular array)
- Tessellation (tiling of a plane using one or more geometric shapes with no overlaps and no gaps)
- Whole number (e.g., 0, 1, 2, 3,...)

Familiar Terms and Symbols

- Addends
- Doubles
- Equation
- Number path
- Number sentence
- Pair
- Rectangle
- Skip-counting
- Square
- Sum
- Tape diagram
- Total
- Unit

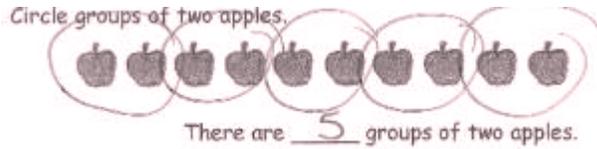
Suggested Tools and Representations

- Counters
- Number path
- Rectangular array
- Square tiles

Lesson 1

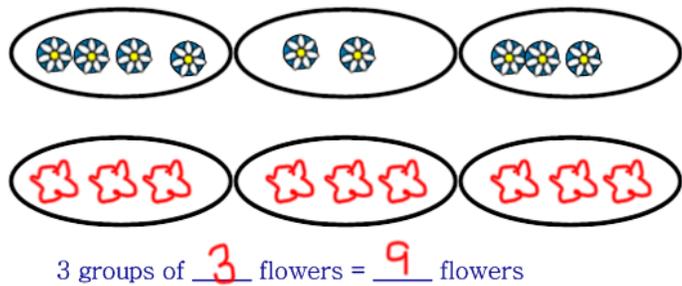
Objective: Use manipulatives to create equal groups.

Groups are made of equal amounts.



We can reorganize

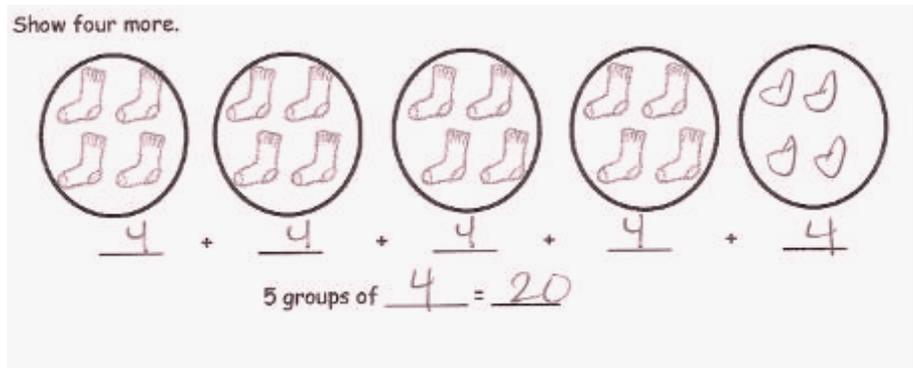
Redraw the flowers to make each of the 3 groups have an equal amount.



Lesson 2

Objective: Use math drawings to represent equal groups, and relate to repeated addition.

We can count groups by using repeated addition.



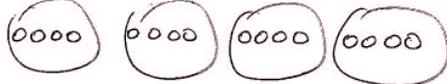
Draw 3 groups of 5 stars. Then write an addition sentence to match.



Lesson 3

Objective: Use math drawings to represent equal groups, and relate to repeated addition.

We can combine addends to simply a long repeated addition equation


$$\begin{array}{r} 4 + 4 + 4 + 4 = 16 \\ \hline \end{array}$$
$$\begin{array}{r} \diagdown \quad \diagup \\ 8 \end{array} + \begin{array}{r} \diagdown \quad \diagup \\ 8 \end{array} = 16$$


$$\begin{array}{r} 2 + 2 + 2 + 2 = 8 \\ \hline 4 + 4 = 8 \\ \hline \end{array}$$

4 groups of 2 = 2 groups of 4


$$\begin{array}{r} 3 + 3 + 3 + 3 + 3 + 3 = 18 \\ \hline \end{array}$$
$$\begin{array}{r} \diagdown \quad \diagup \\ 6 \end{array} + \begin{array}{r} \diagdown \quad \diagup \\ 6 \end{array} + \begin{array}{r} \diagdown \quad \diagup \\ 6 \end{array} = 18$$
$$\begin{array}{r} \diagdown \quad \diagup \\ 12 \end{array} + \begin{array}{r} \diagdown \quad \diagup \\ 6 \end{array} = 18$$

Lesson 4

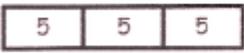
Objective: Represent equal groups with tape diagrams, and relate to repeated addition.

We can use a tape diagram to organize our thinking. Numbers can be used instead of drawing pictures.

b. 

$$\begin{array}{r} 4 + 4 + 4 + 4 + 4 = 20 \\ \hline \end{array}$$

5 groups of 4 = 20

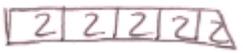
c. 

$$\begin{array}{r} 5 + 5 + 5 = 15 \\ \hline \end{array}$$

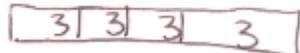
3 groups of 5 = 15

5 groups of 2

10



3 + 3 + 3 + 3 = 12



Lesson 5

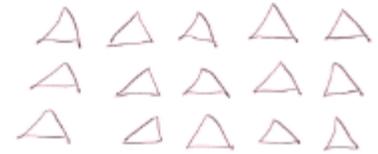
Objective: Compose arrays from rows and columns, and count to find the total using objects.

We can organize groups into arrays.

Arrays are made of rows and columns.

Rows are arranged horizontally and columns are arranged vertically.

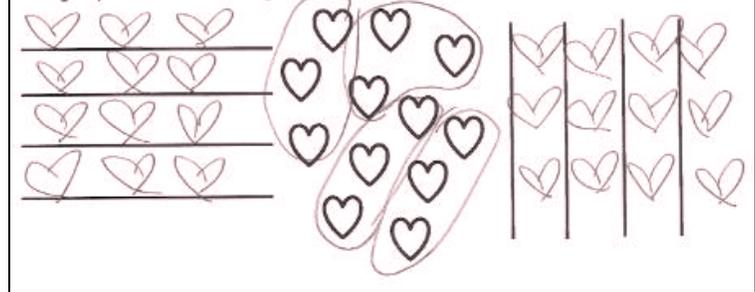
Draw an array with 15 triangles.



Show a different array with 15 triangles.



Circle groups of 3. Redraw the groups of three as rows and then as columns.



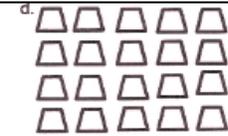
Lesson 6

Objective: Decompose arrays into rows and columns, and relate to repeated addition.

We can identify and count by rows (horizontal groups) and columns (vertical groups).

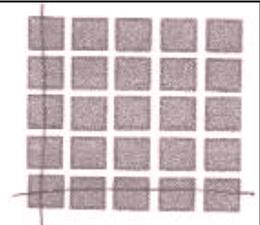


4 rows of 5 = 20
 $5 + 5 + 5 + 5 = 20$



5 columns of 4 = 20
 $4 + 4 + 4 + 4 + 4 = 20$

Use the array of squares to answer the questions below.



a. $5 + 5 + 5 + 5 + 5 = 25$

b. 5 rows of 5 = 25

c. 5 columns of 5 = 25

d. Remove 1 row. How many squares are there now? 20

e. Remove 1 column from the new array you made in 4(e). How many squares are there now? 16

Lesson 7

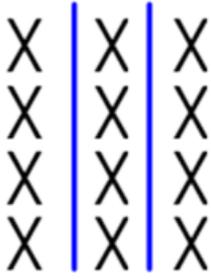
Objective: Represent arrays and distinguish rows and columns using math drawings.

4 rows of 3



$3 + 3 + 3 + 3 = 12$
4 rows of 3 = 12

3 columns of 4



3 columns of 4 = 12
 $4 + 4 + 4 = 12$

Lesson 8

Objective: Create arrays using square tiles with gaps.

Use the array of squares to answer the questions below.



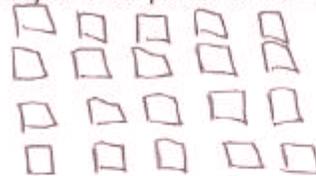
a. There are 5 squares in each row.

b. $\underline{5} + \underline{5} = \underline{10}$

c. There are 2 squares in each column.

d. $\underline{2} + \underline{2} + \underline{2} + \underline{2} + \underline{2} = \underline{10}$

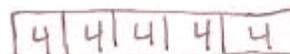
a. Draw an array with 20 squares that has 4 squares in each column.



b. Write an addition number sentence to match the array.

$4 + 4 + 4 + 4 + 4 = 20$

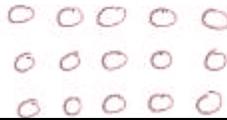
c. Draw a tape diagram to match your addition sentence and array.



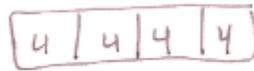
Lesson 9

Objective: Solve word problems involving addition of equal groups in rows and columns.

There are 3 wires and 5 birds sitting on each of them. How many birds in all are on the wires?


$$5 + 5 + 5 = 15 \text{ wires}$$

In a card game, 3 players get 4 cards each. One more player joins the game. How many total cards should be dealt now?


$$4 + 4 + 4 + 4 = 16 \text{ cards}$$

$$4 + 4 + 4 + 4 = 16 \text{ cards}$$

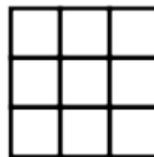


Lesson 10

Objective: Use square tiles to compose a rectangle, and relate to the array model.

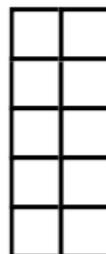
We can make square arrays and rectangular arrays using tiles.

Construct a rectangle of 9 tiles so that the rows are the same size as the columns.



$$3 + 3 + 3 = 9$$

Construct a rectangle with 5 rows of 2 tiles.



$$2 + 2 + 2 + 2 + 2 = 10$$

Lesson 11

Objective: Use square tiles to compose a rectangle, and relate to the array model.

a. Construct an array with 20 squares.

b. Write a repeated addition sentence to match the array.

$$\underline{5+5+5+5=20}$$

c. Rearrange the 20 squares into a different array.

d. Write a repeated addition sentence to match the new array.

$$\underline{2+2+2+2+2+2+2+2+2+2=20}$$

Construct 2 arrays with 10 squares.

a. 2 rows of 5 = 10

b. 5 rows of 2 = 2 rows of 5

Lesson 12

Objective: Use math drawings to compose a rectangle with square tiles.

We can use tiles or draw pictures of rectangular arrays to show our math thinking.

We never leave space between tiles when making an array.

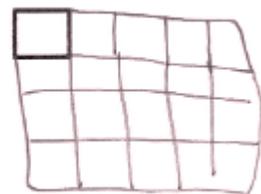
Draw without using a square tile to make an array with 2 rows of 5.



2 rows of 5 = 10

5 + 5 = 10

5 columns of 4.



Lesson 13

Objective: Use square tiles to decompose a rectangle.

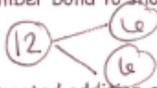
We can break apart a larger array into two smaller arrays. This can be represented using a number bond.

Problem 1:

Step 1: Construct a rectangle with 4 columns of 3.

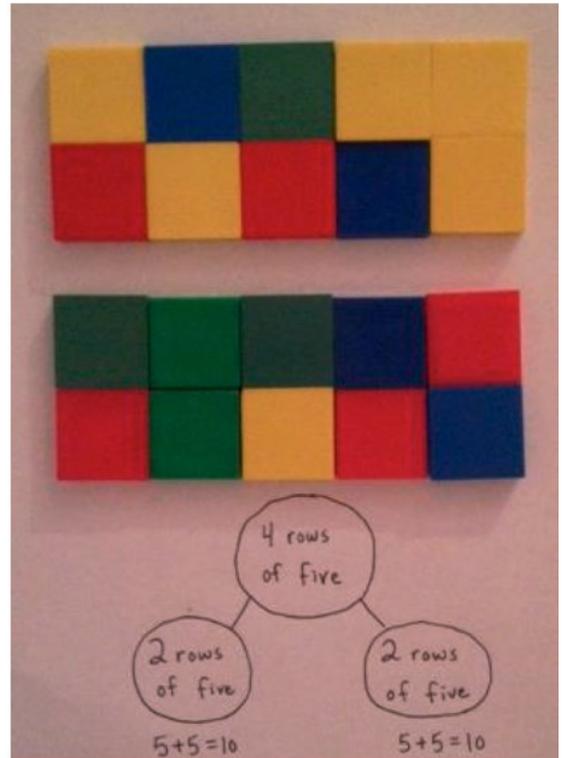
Step 2: Separate 2 columns of 3.

Step 3: Write a number bond to show the whole and two parts.



Step 4: Write a repeated addition sentence to match each part of the number bond.

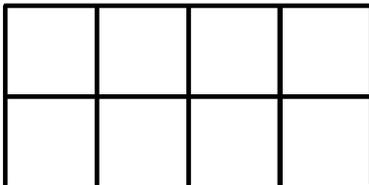
$$3 + 3 + 3 + 3 = 12$$



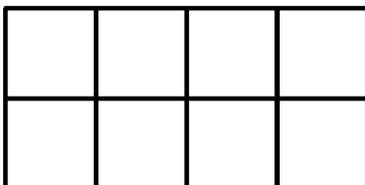
Lesson 14

Objective: Use scissors to partition a rectangle into same-size squares, and compose arrays with the squares.

Rectangle A



Rectangle B



Cut out each row of Rectangle A.

a. Rectangle A has 2 rows.

b. Each row has 4 squares.

c. 2 rows of 4 = 8

d. Rectangle A has 8 squares.

• Cut out each square from both rectangles A and B.

a. Construct a new rectangle using all 16 squares.

b. My rectangle has 4 rows of 4.

c. My rectangle also has 4 columns of 4.

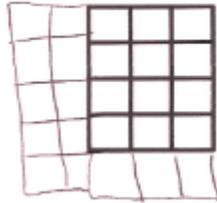
d. Write two repeated addition sentences to match your rectangle.

$$4 + 4 + 4 + 4 = 16$$

Lesson 15

Objective: Use math drawings to partition a rectangle with square tiles, and relate to repeated addition.

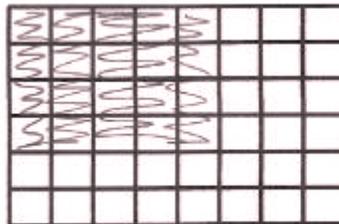
Draw one more row and then two more columns to make a new array.



Write a repeated addition sentence for the new array.

$$5+5+5+5+5 = 25$$

Shade in an array with 5 columns of 4.



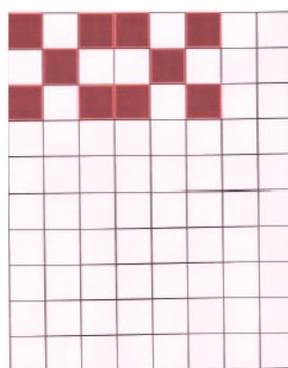
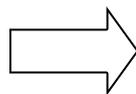
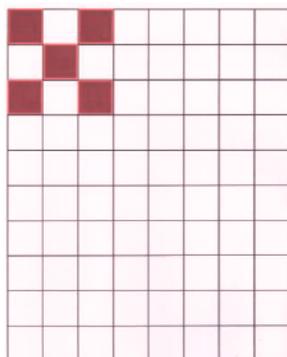
Write a repeated addition sentence for the array.

$$4+4+4+4+4 = 20$$

Lesson 16

Objective: Use grid paper to create designs to develop spatial structuring.

We can repeat a pattern created with tiles



Lesson 17

Objective: Relate doubles to even numbers, and write number sentences to express the sums.

2. Draw an array to match each set.

a. 2 rows of 6

 2 rows of 6 = 12
 $6 + 6 = 12$
 6 doubled is 12

b. 2 rows of 7

 2 rows of 7 = 14
 $7 + 7 = 14$
 7 doubled is 14

c. 2 rows of 8

 2 rows of 8 = 16
 $8 + 8 = 16$
 8 doubled is 16

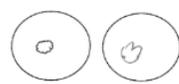
d. 2 rows of 9

 2 rows of 9 = 18
 $9 + 9 = 18$
 9 doubled is 18

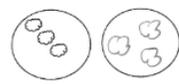
e. 2 rows of 10

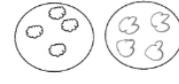
 2 rows of 10 = 20
 $10 + 10 = 20$
 10 doubled is 20

1. Draw to double the group you see and complete the sentences.

a.  There is 1 clouds in each group.
 $1 + 1 = 2$

b.  There are 2 clouds in each group.
 $2 + 2 = 4$

c.  There are 3 clouds in each group.
 $3 + 3 = 6$

d.  There are 4 clouds in each group.
 $4 + 4 = 8$

e.  There are 5 clouds in each group.
 $5 + 5 = 10$

List the totals from Problem 1. 2, 4, 6, 8, 10
 List the totals from Problem 2. 12, 14, 16, 18, 20
 Are the numbers you have listed even or not even? even
 Explain in what ways the numbers are the same and different.
They are all even. They all come from groups of 2 or 2 groups of a number.

Lesson 18

Objective: Pair objects and skip-count to relate to even numbers.

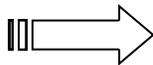
We only get a doubles fact when all of the objects have a partner. If any objects are left over without a partner, it can't be even.

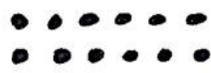
Circle groups of two. Count by 2s to see if the number of objects is even.



a. There are 9 twos. There are 0 leftover.
 b. Count by 2s to find the total:
2 4 6 8 10 12 14 16 18
 c. This group has an even number of objects. True False

Eggs come in cartons of 12. Use pictures, numbers, or words to explain whether 12 is even or not even.



12 is even because the eggs look like this. 
 2 rows of 6 is the same as $6 + 6$ so 12 is even.

Lesson 19

Objective: Investigate the pattern of even numbers: 0, 2, 4, 6, and 8 in the ones place, and relate to odd numbers.

Adding or subtracting 1 to an even number will make it odd. Numbers that have 0,2,4,6, or 8 in the one's place will be even.

000000
00000~~0~~

$$12 - 1 = 11$$

11 is odd because you can't count by 2's to 11.

$$\begin{array}{l} 24 + 1 = 25 \\ \text{even} + 1 = \text{odd} \end{array}$$

$$\begin{array}{l} 24 - 1 = 23 \\ \text{even} - 1 = \text{odd} \end{array}$$

28 even / odd	Explanation: 11 end in 8 so it's even
39 even / odd	Explanation: 8+1 = 9 so it's odd

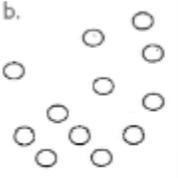
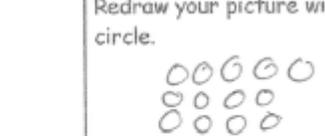
Lesson 20

Objective: Use rectangular arrays to investigate odd and even numbers.

Even + Even = Even $12 + 4 = 16$

Even + Odd = Odd $12 + 3 = 15$

Odd + Odd = Even $11 + 3 = 14$

b. 	Array 	Redraw your picture with 1 more circle. 
There is an even/odd (circle one) number of circles.	There is an even/odd (circle one) number of circles.	There is an even/odd (circle one) number of circles.